MT365 Guide

Contents

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6.1

The purpose of this guide is to give you some idea of the content, philosophy and structure of the module, and to explain how we have designed the various components. We know that you will be keen to get started on the units, but we suggest that you first spend about half an hour on this guide to get some general ideas about the module, and to avoid wasting time and effort later.

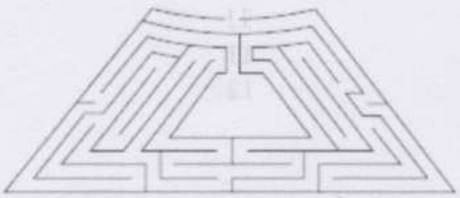
This guide consists of three sections. In Section 1, *Introduction*, we list some typical problems to be solved in the module, and briefly describe the main themes: *combinatorics* and *mathematical modelling*. In Section 2, *Components of MT365*, we discuss the structure of the study units, the multimedia components, assignments and other related topics. Finally, in Section 3, *Content of the module*, we include a brief description of each of the three main areas (Graphs, Networks and Design), a list of units and a diagram showing how the units depend on each other.

MT365 was originally written with computer activities embedded. These activities are now optional, and full marks for the module can be obtained without using the optional module software; you may choose to ignore all reference to these activities and the optional module software.

1 Introduction

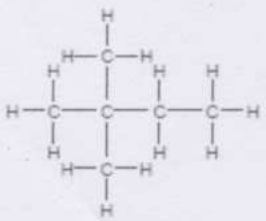
The following are just a few of the many problems that you will look at while studying MT365.

- Suppose that you are stuck in the middle of a maze. Is there any guaranteed method for finding your way out?



Hampton Court maze

- When a spacecraft goes into orbit, it needs to be able to communicate with the Earth. How do you construct a suitable code to enable it to do so reliably?
- If you wanted to drive from Land's End to John O'Groats, how would you find the shortest route?
- If you try to colour a map of Europe in such a way that neighbouring countries are assigned different colours, you will find that only four colours are necessary. Is this true for all maps, or are there maps that need more colours?
- How many molecules are there with the formula C_6H_{14}?



a molecule

- Is it possible to tile a floor with a combination of regular 12-sided, 6-sided and 4-sided tiles?
- What is the most efficient way of bracing a given rectangular framework to make it rigid?
- How can you design a gas pipeline network at minimum cost, if there are restrictions on the amount of flow, pipe diameters, and cost per kilometre?

Although these problems may seem very diverse at first sight, they can all be expressed as problems involving the arrangements of certain objects and the relationships between these objects. By developing general methods for tackling problems of this kind, we will show both how to solve such problems and also how to spot connections between problems that may appear at first sight to have little in common. By understanding the underlying reasons for these connections, you will gradually gain further insight into the nature of the original problems and their solutions.

The branch of mathematics that deals with such arrangement problems is known as combinatorial analysis, or combinatorics; the development of this subject is one of the themes of MT365. We split the subject of combinatorics into three interrelated areas – Graphs, Networks and Design – each of which has four units devoted to it. We will develop general methods for solving a variety of combinatorial problems, and then apply these methods to the above problems, and many others.

Many of the situations that we examine in this module arise from important practical problems in technology and science. Indeed, much of the impetus given to the subject has arisen out of the need to solve particular problems in industry involving network analysis or operational research. By representing these problems in combinatorial terms and applying combinatorial techniques of the kind discussed in this module, it has been possible to make substantial savings in cost and time. This brings us to another theme of MT365 – mathematical modelling.

In MT365, the mathematical modelling process involves reformulating a problem in such a way that it can be approached by combinatorial techniques. This is not always easy since the way in which this modelling is carried out, and the degree to which the mathematical model faithfully represents the original problem, vary considerably from problem to problem. Throughout, we emphasize not only the modelling process itself, but also its limitations.

So the two main themes of MT365 are the development of combinatorics as a subject in its own right, and the modelling of practical situations. The primary interest of the mathematician may well be the former, whereas for the technologist and scientist the problems themselves may well provide the main interest. We have tried to integrate these two approaches, since we believe that theory and practice are too interrelated to be separated successfully.

We hope that the module will prove of interest to everyone – whether technologist, scientist or mathematician. For the mathematician it will provide an opportunity to see mathematics in action in solving worthwhile problems, and for the technologist and scientist it will show the importance and usefulness of developing a mathematical framework that can be used to interrelate different problems, and provide means for solving them.

The subject that you are about to study is an exciting one. Although its roots go back a long way, it is a modern subject in which substantial advances are being made all the time. It is likely to play an ever-increasing role in the years to come, and MT365 is designed to give you the necessary background to understand these future developments.

2 Components of MT365

MT365 is a 30-credit module, and consists of fourteen units. The Introduction unit is not to be studied in great depth, but to be treated as a taster with full understanding coming later when the topics are covered in the units. The Conclusion unit is not directly assessed; it summarises and brings together many ideas of the module, so it is worth skimming through towards the end of the module to see if anything helps with your revision, then to read it properly after the end of the module. Each of the twelve main units is intended to take approximately 10–12 hours of study time, and consists of a study text and some assignment questions. Several units also involve video tracks, audio sequences and other supplementary material. There is no set book. There is a three-hour examination at the end of the module.

MT365 has been presented a good number of times and so has an extensive collection of past examination papers – these should be beneficial for your revision. In addition, through experience, a few areas have been highlighted where an additional explanation or example is helpful, or where a slightly different method of presentation of an algorithm has worked well. The module team have therefore produced some additional resources and have commissioned an eTutorial for each of the twelve main units of the module, plus one for revision. These can be found on the module website.

2.1 Module website

The module website, accessed via the link on your StudentHome web page, is an important resource for MT365. The *Study planner* will tell you the cut-off dates for the assignments, which units have associated video tracks, audio sequences, and/or other additional resources. You should study this planner carefully.

Any important notices for the presentation will be posted to the *News* section of the module website.

The *Assessment* section, a link to which can be found towards the top of the module website, gives access to the assessment materials.

The *Resources* section of the module website contains errata and other important information, electronic versions of the study materials, the Graph Database and the optional module software. The recorded eTutorials, one on each unit, plus the associated slides and questions, can also be found here.

The module website also hosts the *module forums*. These forums are moderated and can be useful for asking module-related questions, although questions on current assignment questions are not permitted.

2.2 Study units

Most units have five sections, each of which is designed to be approximately an evening's work. Each section ends with a list of objectives that provides a useful checklist to which you should refer after reading the section.

Study guide and Introduction

Each study unit begins with a *Study guide*, followed by an *Introduction*.

The *Study guide* includes a *Plan of the unit* – a diagram giving the titles of the various sections, indicating how these sections depend on one another; television, and audio-tape symbols are included in the appropriate places in the *Plan* and also in the text. These symbols relate to the video tracks and audio sequences. Computer symbols are also included and these relate to the optional computer activities, so you may prefer to ignore these. The *Study guide* also gives guidance on how to plan your study of the unit. In some cases you may wish to study the sections in a different order from that suggested (for example, if you are working on a bus or train), and the *Plan* and *Study guide* will indicate whether this is possible.

The *Introduction* gives a brief overview of the unit, and outlines the content of each section.

Problems

You will be working through many problems while studying the texts, so you will need a pencil and paper handy. The solutions to the problems are provided at the back of each unit. You will gain most benefit if you attempt each problem before looking at our solution. You should certainly read each solution before proceeding, even if you are confident that your solution is correct, as our solutions sometimes contain additional comments or material relevant to the subsequent text, and indicate whether other solutions are possible.

Exercises

Each unit contains a set of exercises, similar to the problems, which you can work through to gain extra practice, or use for revision purposes. The solutions are given in the section following the exercises.

Further reading

Near the end of each unit is a list of relevant further reading material, in case you wish to study some of the topics in greater depth.

Errata

A list of known errata can be found in the *Resources* section of the module website. We advise that you correct *all* the known errata in your study materials *before* starting the module.

We acknowledge the large number of errata for the module, in particular the age of a couple of the errata (it was an oversight that the dates that Hamming and Golay died were not updated when the module was last reprinted), but we are unfortunately not in a position to be able to correct them in the study materials at the present time. We hope that the majority of errata have now been spotted and so the likelihood of new errata arising is low.

2.3 The Graph Database

The Graph Database can be found in the *Resources* section of the module website (in interactive form and as a PDF file). It contains 1252 standard graphs, listing their number of vertices n, number of edges m, and degree sequence. Each graph is referred to by its position in the database, so for example, graph G10 is the tenth in the database and has $n = 4$, $m = 2$ and degree sequence $(0, 1, 1, 2)$. Some assignment questions refer to graphs in the Graph Database.

The Graph Database is also part of the optional module software. This optional module software is available to download from the module website. The associated *Computer Activities* booklet is also found here; it contains information on the optional computer activities.

2.4 Video tracks

The video tracks for this module were originally recorded as seven television programmes; they are now provided on DVD.

In some units, we use the programmes flexibly – you can watch them at any time during your study of these units.

In other units, the programmes are more tightly integrated with the rest of the unit, and you should try to watch the programme at the appropriate stage. Since the programmes are an integral part of the module, they may be assessed in the tutor-marked and computer-marked assignments. You are advised to look out for any pertinent assessment material before watching each programme, and, if possible, try the assignment immediately afterwards.

The *Television Notes* contain a summary of each programme. You should refer to the notes before watching each programme; they may also contain material to be read after watching the programme and, occasionally, extra material not directly linked to the programme.

Transcripts of the television programmes are available on the module website.

2.5 Audio sequences

Several units (usually the Networks units) have audio sequences associated with certain sections. These were originally recorded as audio-tape sections; they are provided on CD and are also available from the module website. You will need to listen to the relevant sequences while working on these sections, and will need to follow the relevant frames in the *Audio-tape Notes*.

You may need to stop the audio to tackle a problem or to answer a question, so you should have a pencil and paper to hand. The points at which you should pause are indicated by an audible signal, although you may pause at any time.

Transcripts of the audio sequences are available on the module website.

2.6 Handbook

There is an MT365 *Handbook* that summarizes the main results of each unit. You may take the Handbook into the examination room, and you may annotate it in any way you see fit, but you may not insert extra pages. We suggest that you write anything that you might want to add on Post-its in the first instance. Then, before the examination, copy into your Handbook the contents of the Post-its that you have used regularly. (You will not be allowed to take the Post-its into the examination.) We recommend that you use the Handbook for reference from the start of the module, so that you are thoroughly familiar with it when you come to the examination.

2.7 Assessment

The module is assessed by means of four tutor-marked assignments (TMAs), four computer-marked assignments (iCMAs), and a three-hour examination at the end of the presentation.

The cut-off dates for the TMAs and iCMAs are given in the *Study planner* on the module website, and in the relevant *Assignment*.

Your overall score

Your overall score for the module will be calculated as $0.2\times$ your continuous assessment score $+ 0.8\times$ your examination score. Your continuous assessment score is the weighted average of your TMA and iCMA scores. To pass the module you will usually need to achieve an overall score of at least 40%.

It is very important that you engage fully with the TMAs and iCMAs, as they help to develop your learning and understanding of the topics covered and provide you with tutor feedback on your work. **Taking time to work through in detail the TMA and iCMA content, and the feedback you subsequently receive, in detail is vital for successful examination preparation.**

The contribution of each assessment score to the overall score is given in the table below.

Assessment	Units covered	Contribution to overall score
TMA 01	G1, N1, D1	3%
TMA 02	G2, N2, D2	3%
TMA 03	G3, N3, D3	3%
TMA 04	G4, N4, D4	3%
iCMA 41	Intro, G1, N1, D1	2%
iCMA 42	G2, N2, D2	2%
iCMA 43	G3, N3, D3	2%
iCMA 44	G4, N4, D4	2%
Examination	G1–4, N1–4, D1–4	80%

Based on your overall score you will be awarded one of the following results:

- Pass 1
- Pass 2
- Pass 3
- Pass 4
- Fail

Normally, for a Pass 4 result you must achieve an overall score of at least 40.

Tutor-marked assignments

The *Assessment* section on the module website gives access to the TMAs; they each contain details on how to submit them for marking, and how to interpret some of the key words used in questions, such as 'state', 'determine' and 'find'.

Before completing your first TMA, read the following notes carefully.

- All the questions in each TMA are assessed.
- Handwritten solutions, or scanned handwritten solutions to TMAs are perfectly acceptable, provided that they are legible. You may word-process your work if you wish, but it can be time consuming to produce the mathematical symbols and layouts that you will need, and it may be better to spend the time on the mathematics. You should use dark ink, as your work may be photocopied or scanned for monitoring of consistent marking by tutors.
- You should number all the pages of your work on a TMA, and give your name and personal identification (PI) number on each page.
- The number of marks assigned to each part-question is given in the TMA, and should give you an indication of the relative difficulty or importance of each part-question. Do not write several pages on a part-question assigned 1 mark, or two lines for a part-question assigned 10 marks!
- If you use a definition, result or theorem to get from one line to the next, you should quote its number, or the page of the *Handbook* or unit on which it appears. When using a theorem, always demonstrate that the conditions of the theorem are satisfied.
- If you have difficulty with any question, seek help and advice from your tutor.
- If you have difficulty in meeting a cut-off date, contact your tutor as early as possible.
- Tutors are asked to comment fully on each question and to use every opportunity to teach on the basis of your solutions. Always try to write something for each question, as your tutor will not know how to help you if you just leave a blank space. Writing 'I'm stuck here' or 'I need to be able to prove so-and-so, but I can't' will help your tutor to help you. Please leave enough space between question parts for your tutor to add comments.

- When you get an assignment back, spend some time studying your tutor's comments, and rewrite any incorrect or incomplete answers along the lines suggested. You will find such time well invested when you come to use the TMAs for revision purposes. We regard these comments by your tutor as a major part of the teaching of the module.
- For questions permitting the use of the optional module software, you may submit printouts to supplement your answers if you wish. However, you must annotate your printout so that it is clear how your answer arises from it.

Computer-marked assignments

The *Assessment* section on the module website gives access to both the CMAs (containing the questions) and the iCMA system for submitting your answers.

Before completing your first CMA, read the following notes carefully.

- All the questions in each CMA are assessed.
- Each CMA contains instructions for submitting your answers.
- You can save your answers on the iCMA system without submitting your CMA, so you can work on an assignment over a period of time. Always remember to check all your answers before you finally submit your CMA.
- In MT365 there are no penalty marks for wrong answers. It is therefore better to make an intelligent guess than to leave a question unanswered.
- The correct answers and feedback on the questions will be provided through the iCMA system after the cut-off date, once your assignment has been marked.

Essay writing

This module includes elements that are designed to help you to develop your skills in understanding mathematical ideas and in communicating them to a general audience. Two of the assignments contain one part-question that requires you to write a short essay. Furthermore, you may answer at most one Part 2 essay question on the final examination paper.

The idea of the essay is to explain a small portion of the material from the module to a more general audience. You may assume that the intended reader is another undergraduate student who is studying mathematics modules, but who has no specific knowledge of this one.

Some advice on writing essays for this module is provided on the module website as an additional resource.

Final examination

The three-hour final examination has two parts. Part 1 consists of routine short-answer questions, and is designed to test your coverage of the material. Part 2 consists of longer questions, with a certain amount of choice, and is designed to test a deeper understanding of the material.

There are many topics covered in the module, and you are not expected to remember them all, so we have produced a *Handbook* (see Section 2.6), which you may annotate during the presentation and take into the examination.

A *Specimen Examination Paper* and *Solutions* are provided on the module website. These will give you an idea of the format of the examination, and the types of question that will be set. You should study both the instructions on the specimen paper and the solutions carefully so that you know what sort of answers you will need to write down in order to get full marks. Past examination papers are also available from the Open University Students Association (OUSA).

Calculators are not permitted in the final examination.

2.8 Workload

Your personal circumstances may not permit you to study every part of every unit. If this happens, you should concentrate on the parts of the unit that are covered by the assignments. It is more important to keep up than to understand every single detail of each unit before proceeding to the next!

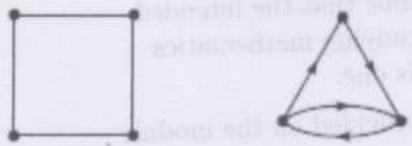
If there are problems at home or at work that you think will affect your ability to complete an assignment before the cut-off date, then you should contact your tutor immediately.

3 Content of the module

The module comprises an *Introduction* unit, four units on each of the topics *Graphs*, *Networks* and *Design*, and a *Conclusion* unit. We sometimes refer to a particular trio of units such as *Graphs 1*, *Networks 1* and *Design 1* as 'Block 1'. The three main areas are as follows.

3.1 Graphs

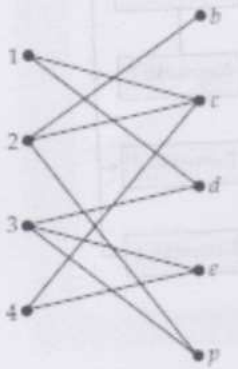
In MT365 the word 'graph' usually refers to a diagram of points interconnected by lines, rather than to a picture representing a function. The points may correspond to towns on a map, atoms in a molecule, people, and so on; the lines connecting the points may correspond to roads between the towns, bonds between the atoms or friendships between the people. Digraphs (short for 'directed graphs') are like graphs, except that each line has a direction indicated by an arrow. A digraph may be used, for example, to represent a one-way street system.



graph digraph

3.2 Networks

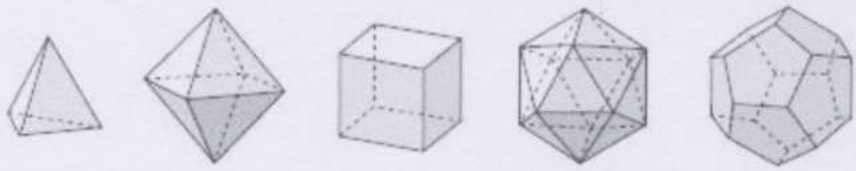
How can you find the shortest route between two towns on a road map? How can a manufacturer send the required amount of a commodity to a number of markets at minimum cost? How do you make the best assignment of a number of candidates to a number of jobs? These are examples of the types of network problem that you will meet in MT365.



assignment

3.3 Design

These units cover four main topics: geometric design, kinematic design, design of codes and block designs. The examples considered include plane tilings, the design of a robot manipulator or an aeroplane simulator, the design of error-correcting codes such as those that have been used in interplanetary space probes, and the design of experiments.

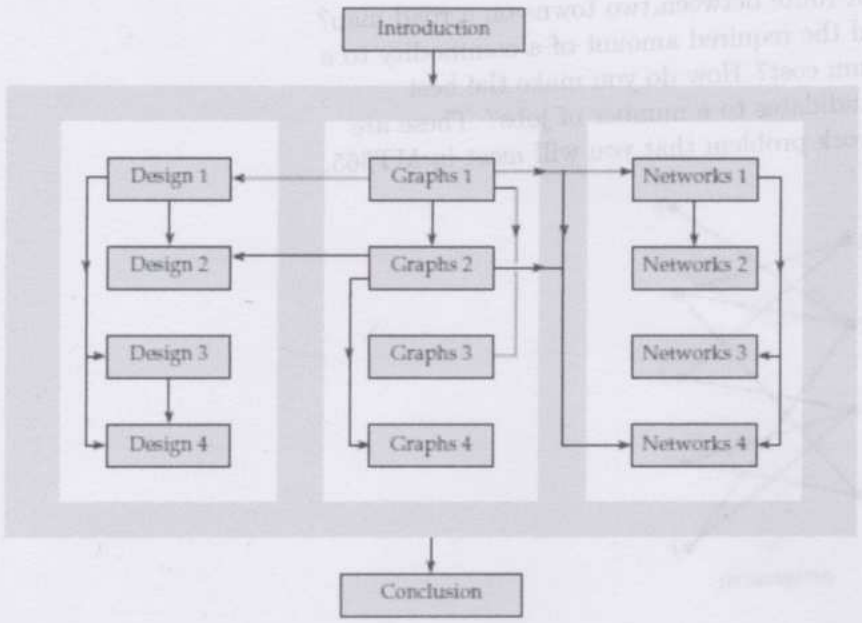


tetrahedron octahedron cube icosahedron dodecahedron

3.4 List of units

Introduction			
Graphs 1	Graphs and digraphs	TMA 01	CMA 41
Networks 1	Network flows		
Design 1	Geometric design		
Graphs 2	Trees	TMA 02	CMA 42
Networks 2	Optimal paths		
Design 2	Kinematic design		
Graphs 3	Planarity and colouring	TMA 03	CMA 43
Networks 3	Assignment and transportation		
Design 3	Design of codes		
Graphs 4	Graphs and computing	TMA 04	CMA 44
Networks 4	Physical networks		
Design 4	Block designs		
Conclusion			

3.5 Plan of the module



MT365 Non-examined topics

This list prepared by the module team should save you some time at the end of the presentation.

The **Introduction** unit is not directly assessed; that is, there are no examination questions relating to this unit. Many of the topics introduced in this unit occur in later units and may therefore be assessed as part of that unit. The **Conclusion** unit is not assessed.

The following topics will **NOT** be tested in the examination.

Graphs 1

All case studies: Subsections 1.5, 2.4, 3.5, 4.3.

Four cubes problem, Chemistry, Music, Social Networks, Diagram tracing, Knights Tour, Gray codes, Ecology, Social networks, Rotating drum, Ranking in tournaments, Interval graphs, Archaeology, Genetics, Markov chains.

Graphs 2

Examples of trees and Modelling with trees: Subsections 1.2, 1.3.
Postscript on enumeration problems: Subsection 2.4.
Gas pipeline networks: the whole of Section 5.

Graphs 4

Branch and bound method for the Travelling Salesman Problem: Subsection 5.3.

Networks 1

Reliable telecommunication networks: Subsection 1.4.

Proof of Menger's Theorem: Subsection 3.4.
Networks with upper and lower capacities: the whole of Section 4.

Networks 2

Eulerian and Hamiltonian Digraphs: Subsections 1.2 and 1.3.

Splitting into cycles, Finding Hamiltonian cycles, Latin matrix multiplication '#'.

Networks 3

Modified Marriage Theorem and Proofs of the Marriage Theorem: Subsections 1.2 and 1.3.

Networks 4

Tellegen's Theorem: Subsections 2.6.
Solving the matrix equation (Gaussian elimination): Subsection 3.2.
Finding the state equations (using Gaussian elimination or directly): in Subsection 3.3.
(*Although the matrix equation* $\mathbf{Hx} = \mathbf{y} + \mathbf{K}\dot{\mathbf{x}}$ *may be included and you may be asked to specify the state variables.*)

Design 1

Tilings at the Alhambra: Section 3.
Finite projective geometry: Subsection 5.3.
(*Although projective planes have been included as a* Design 4 *topic.*)

Design 2

Other planar and spatial systems: Subsection 4.6.
Flight simulators and robots: the whole of Section 5.

Design 3

The Golay codes and the Mariner 9 Code: Subsections 4.3 and 4.4.
Compact discs: the whole of Section 5.

TV Notes and Video programmes

TV programmes 1, 2, 3, 4, 6.
(*TV5, The four-colour theorem is the only programme that may be assessed directly, although the topics covered in TV7, Transporting flowers, may occur as parts of the relevant units.*)